

2021**MATHEMATICS — HONOURS****Paper : DSE-B-1****(Discrete Mathematics)****Full Marks : 65***The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words
as far as practicable.*

1. Answer the following multiple choice questions (MCQ) in which only one option is correct. Choose the correct option with proper justification if any. 2×10
- (a) Consider the lattice $S = \{1, 2, 3, 4, 6, 12\}$, the divisors of 12 ordered by divisibility. The complement of 3 is
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|---------|---------|
| (i) 1 | (ii) 12 |
| (iii) 2 | (iv) 4. |
- (b) The number of unit element of the ring Z is
- | | |
|---------|---------|
| (i) 2 | (ii) 1 |
| (iii) 3 | (iv) 0. |
- (c) The multiplicative inverse of 7 in Z_{11} is
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|---------|---------|
| (i) 3 | (ii) 8 |
| (iii) 6 | (iv) 2. |
- (d) Let G be a connected graph with 10 vertices. If G is a tree, then the sum of the degrees of the vertices is
- | | |
|----------|----------|
| (i) 10 | (ii) 18 |
| (iii) 20 | (iv) 22. |
- (e) The maximum number of edges in a simple disconnected graph G with 10 vertices and 5 components is
- | | |
|----------|----------|
| (i) 10 | (ii) 15 |
| (iii) 25 | (iv) 50. |
- (f) The maximum and minimum heights of a binary tree with 25 vertices are respectively
- | | |
|-------------|-------------|
| (i) 13, 12 | (ii) 15, 10 |
| (iii) 17, 4 | (iv) 12, 4. |

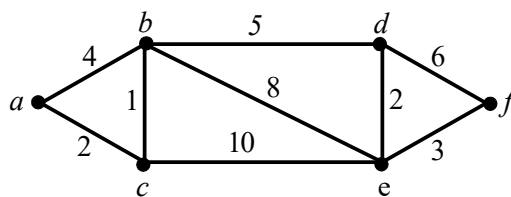
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- (g) The remainder when $2 \times 98!$ is divided by 101 is
- (i) 0 (ii) 1
 (iii) 97 (iv) 100.
- (h) A graph has 15 vertices and 20 edges. The least number of edges to be removed from the graph to make it a tree is
- (i) 13 (ii) 5
 (iii) 19 (iv) 6.
- (i) What is $\sigma(180)$?
- (i) 542 (ii) 544
 (iii) 546 (iv) 548.
- (j) Given any five points inside of a square of side 2, then there exists two points within a distance of atmost
- (i) 1 (ii) 2
 (iii) $\sqrt{2}$ (iv) $\frac{1}{\sqrt{2}}$.

Unit-1

Answer *any five* questions.

2. A connected graph contains an open Eulerian trail if and only if it has exactly two vertices of odd degree. — Prove it. 5
3. If G is a connected planar graph with $n(\geq 3)$ vertices, e edges and no cycle of length 3, then prove that $e \leq 2n - 4$. Hence show that $K_{3,3}$ is non-planar. 3+2
4. Using Dijkstra's algorithm, find the shortest path from the vertex a to f in the following graph. 5



5. Prove that the number of vertices in a binary tree is always odd. Find also the number of pendant vertices of a binary tree with n vertices. 2+3
6. Prove that for any graph G with $n > 2$ vertices must have two vertices of same degree. 5
7. (a) Give an example of a partially ordered set (L, \leq) which is a lattice.
 (b) Prove that in a bounded distributive lattice (L, \vee, \wedge) if an element $a \in L$ has a complement, then it is unique. 2+3

8. Let $n \in \mathbb{N}$. Then prove that there exist two positive integers a and b such that $n^a - n^b$ is divisible by 10. 5
9. Find the number of edge disjoint Hamiltonian cycle in the complete graph K_{11} . 5

Unit-2

Answer *any four* questions.

10. Prove that both the functions Tau (τ) and Sigma (σ) are multiplicative functions. 5
11. If p be prime, then $(p-1)! \equiv -1 \pmod{p}$. 5
12. (a) For any two positive integers a and n with $a > n$, show that $n \mid \varphi(a^n - 1)$.
 (b) Find the sum of all positive integers which are less than 2022 and prime to 2022. 2+3
13. Solve the quadratic congruence $x^2 + 7x + 10 \equiv 0 \pmod{11}$. 5
14. A certain integer between 1 and 1000 leaves the remainder 1, 2, 6 when divided by 9, 11, 13 respectively. Find the integer with the help of chinese remainder theorem. 5
15. (a) Prove that for all integer $n > 1$, $n^4 + 4$ is composite.
 (b) If a is prime to b , prove that $a + b$ is prime to ab . 2+3
16. (a) Define Fermat's number. Give example.
 (b) Prove that product of the first n Fermat's number is $2^{2^n} - 1$, by the method of induction. 2+3
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